Transducer Function

$$H(s) = \frac{1}{S_{21}} \equiv \frac{1}{2}\sqrt{\frac{R_L}{R_G}}\frac{E_G(s)}{V_2(s)}$$

$$|H(j\omega)|^2 = \frac{P_{\max,gen}}{P_2} = \frac{1}{|S_{21}(j\omega)|^2}$$

$$\rho_1(s) = \frac{Z_1 - R_1}{Z_1 + R_1} = S_{11}(s)$$

$$|\rho_1(j\omega)|^2 = \frac{P_{refl}}{P_{\max,gen}} \qquad \text{with } P_{refl} = P_{\max,gen} - P_1$$

$H(s), K(s), \rho_1(s)$ are all rational functions.

$$K(s) = \frac{S_{11}(s)}{S_{21}(s)} = H(s)\rho_1(s)$$

$$|K(j\omega)|^2 = \frac{P_{refl}}{P_2}$$

zeros: unity gain frequencies

poles: transmission zeros

If the two-port is lossless $P_1 = P_{\max,gen} - P_{refl} = P_2$
And $|H(j\omega)|^2 - |K(j\omega)|^2 = 1$

As shown later, this leads to

$$H(s)H(-s) = K(s)K(-s) + 1 \qquad (*) \text{ Feldtkeller equation}$$

From which $H(s)$ can be obtained. For active two-ports, it may still be convenient to use $(*)$, although the power relations are no longer valid.

C_n dimension $(\text{rad}/s)^{-n}$, so $C_n \Rightarrow \omega_0^{-n}$

$K(s) = C_n s^n \Rightarrow (s/\omega_0)^n \triangleq S^n$

S : normalized complex freq. variable

$\alpha = 10 \log_{10}\left[|s/\omega_0|^2 + 1\right] \rightarrow 10 \log_{10} 2 \cong 3\,dB$

for $s = j\omega_0$, so $\omega_0 \equiv \omega_{3dB}$

In terms of S,

$H(S)H(-S) = (-1)^n S^{2n} + 1.$

$H(S)$ must have its zeros in the closed LHP : strictly Hurwitz polynomial.

All coefficients must be > 0.

For $n = 2$, $H(s) = a_2 S^2 + a_1 S + a_0$

$(a_2 S^2 + a_1 S + a_0)(a_2 S^2 - a_1 S + a_0) = S^4 + 1$

$a_2 = 1$, $a_0 = 1$

$2 a_0 a_2 S^2 - a_1^2 S^2 = 0 \rightarrow a_1 = \sqrt{2}$

For $n = 3$, $a_3 = a_0 = 1$, $a_1 = a_2 = 2$ (show!)

Becomes messy for high $\underline{n}$.